

OF

AN UPFLOW ANAEROBIC SLUDGE BLANKET REACTOR

A Thesis submitted in partial fulfilment of the requirements for the degree of

MASTER OF TECHNOLOGY

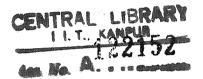
Ву

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CERTIFICATE

This is to certify that the present work Modelling of an Upflow Anaerobic Sludge Blanket Reactor has been carried out under my supervision and has not been submitted elsewhere for a degree.

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ABSTRACT

An Upflow Anaerobic Sludge Blanket (UASB) reactor has been modelled as two mixed reactors (bed and blanket) in series with bypassing and intermixing streams. The model contains mathematical description of the dynamic behaviour and the distribution of both the substrate and the sludge in the reactor, and a quantification of the Monod kinetics of the anaerobic conversion of the waste. The mass balances in the bed and blanket give four first order differential equations which are integrated over a range with suitable initial conditions, using a variable order, variable step method implementing the Backward Differentiation formulae.

The substrate concentration decreases with time and reaches a steady state value in about 5 to 8 days time for the empty bed hydraulic retention time of 15 hours. The microorganisms concentration first increases and then decreases with time because of low substrate concentration in the blanket. To overcome high concentration of microorganism, a part of the sludge is to be removed from the bed and blanket once or twice in a day.

INTRODUCTION

Anaerobic digestion processes occur in many places where organic material is available and presence of oxygen is low as in stomach of ruminants, in marshes, sediments of lakes and ditches, municipal landfills, or even sewers. For a long time these processes have been used by man for the stabilisation of wastes and for the production of methane, a valuable source of energy. Traditional, Chinese and Indian digesters, septic tanks, Imhoff tanks, anaerobic ponds and sludge digestors are typical examples. These traditional systems are defined as low rate systems because they are not based on an understanding of the underlying biotechnological processes and because they require long digestion times and thus large volumes.

More recently, since the seventies attempts to save energy and land have put emphasis on anaerobic treatment as an advanced biotechnology. High rate Upflow Anaerobic Sludge Blankettechniques have evolved for this purpose.

ANAEROBIC MICROBIOLOGY

Anaerobic treatment involves biological processes in which organic material is degraded and biogas composed of mainly methane and ${\rm CO}_2$ is produced. These processes take place in the absence of oxygen. Usually the anaerobic pathway of degradation of organic matter is divided into four steps:

1. Hydrolysis

Proteins fats and polymers are converted to monomers by exoenzymes of micro organisms. This step is in most cases, notably with sewage as a substrate, rate limiting for the overall process and is very sensitive to temperature.

2. Acidification

Formation of alcohols, volatile fatty acids and CO2.

3. Acetogenesis

Formation of acetic acid, hydrogen and CO₂.

4. Methanogenesis

Formation of methane from carbondioxide and hydrogen, and acetic acid.

For anaerobic bacteria this process is energetically very in efficient. A large part of the energy residing in the substrate is liberated in the form of methane and therefore anaerobic bacteria grow relatively slowly compared to aerobic bacteria. At the same time this aspect is the most important feature of anaerobic degradation processes as a waste treatment process, namely

- the energy input of the system is low, as no energy is required for oxygenation,
- the degradation of waste material leads to the production of a valuable source of energy, namely methane,
- the slow growth of the bacteria results in a low production of excess sludge,
- the slow growth of the bacteria also means a low nutrient requirement.

The employment of high rate anaerobic reactors requires some process control, since the methanogenesis is sensitive to the pH: a low pH inhibits the process. If organic loading suddenly increases, and if sufficient chemical buffer capacity would be lacking, the acidification can speed up and lower the pH, thus stopping the methanogenesis and lower fatty acids will accumulate. The reactor may then become irreversibly disturbed and needs renewed startup.

UASB DEVELOPMENT

In late fifties Coulter [1] made a good attempt for development of upflow anaerobic sludge blanket reactor system. Modified anaerobic contact process was claimed as sludge blanket system, where upward movement of liquid waste through a dense blanket of anaerobic sludge was reported. In seventies Stander et al and Cillie et al. (1969), again tried this method for treatment of waste water. Due to heavy sludge washout in effluent, sludge retention was considered to be the primary limitation of process.

Lettinga et al [2] reported slightly modified UASB reactor for better waste stabilisation. There are three basic ideas underlying the process, namely:

- (a) The anaerobic sludge obtains and maintains super settling characteristics if chemical and physical conditions favourable to sludge flocculation and to the maintenance of a well flocculated sludge are provided,
- (b) A sludge blanket or bed may be considered as separate more or less fluid phase with its own specific characteristics in a well established sludge blanket frequently forming a rather stable phase, capable of withstanding relatively high mixing

forces; and

(c) the washout of discrete sludge particles released from the sludge blanket can be minimised by creating a quiescent zone within the reactor, enabling the sludge particles to flocculate, to settle and or to be entrapped in a secondary sludge blanket.

THE PROCESS

The waste water to be treated is introduced into the reactor at the bottom [3]. Inert media are generally absent from the system and the biomass is maintained in suspension by gas bubbles. The bacteria develops as a flocculant mass in an upward flowing waste stream. The microbial blanket is retained by its own mass and by baffles or screens forming the settler unit in the upper portion of the reaction vessel, whilst gas and liquid escape from the top of the tank. As dissociation of the bacterial mass does occur to some degree, organisms are lost in the outflow, but the mean detention time of the bacteria is protracted enough to allow the growth of a dense mass of methanogenic microorganisms although liquid detentions times are low.

In the UASB reactor, the biomass is present as compact grains or granules which develop under the continuous upflow conditions. One of the two fundamental design principles for maintenance of high sludge retentions in the UASB reactor is founded on sludge with improved sedimentations properties. Sludge settleability characteristics improve if mechanical agitation of the sludge bed is minimal or absent. Above the sludge bed the sludge blanket develops: the blanket consists of smaller grains, flocs and gas bubbles and ranges from dense and granular particles with high

settling velocities near the base to lighter, less dense grains higher in the blanket. The UASB system is mixed by hydraulic upflow and rising gas bubbles, and improved agitation can be achieved by the use of several influent ports to deliver feed to the vessel. COD removal occurs throughout the bed and blanket.

second main design principle of the UASB is the installation of a gas/solid separating or settling device in the upper part of the reaction vessel. The smaller particle size and flocculation characteristics of the blanket zone give rise to a settling rate inferior to that found in the bed. Thus to prevent retention of the blanket sludge, the applied liquid velocity in the settler unit should be relatively low: higher velocities tend to produce unacceptable sludge losses. lower velocities, however, allow the accumulation of any sediment. from the influent waste water in the reactor and in the sludge, resulting in decreased sludge activity. The UASB is not, effective treatment process for therefore an waste containing high suspended solids concentrations and organics in the waste stream must posses a proportionately high degree of solubility.

Mixing in the UASB promotes a greater degree of contact between the waste water and the bacterial flora, but internal or external separation of solids and liquid/gas is still necessary. Ensuring even flow distribution and avoiding influent bypass presents problems, as does start up.

The settling process allows the reactor solids to be recycled back to the system; long (sludge retention time) SRTs are hence maintained even with short (hydraulic retention time) HRTs and in

consequence, the size of the reactor can be significantly reduced. The bacterial population of the system can also develop more quickly in a regime incorporating recycle, permitting improved stability and reducing start-up times after interruption. The quantity of biomass in the reactor can be maintained at a constant level by means of the periodic wasting of small amounts of sludge.

One of the most serious limitations of the sludge blanket process is the considerable time (4-8 weeks using seed sludge) involved in the initiation of digestion. The washout of sludge during the initial phase of operation is significant and the consequent loss of net bacterial growth with the effluent engenders a depression in the retained sludge volume and stagnation in the gas production phase of the reaction. Composition of waste water plays a major part in the operation of UASB reactors.

Initial seeding with an active digester sludge is a prerequisite for effective start up of a UASB reactor, as are prevention of pH and toxic shocks, gradual loading increases and extended HRTs.Start up, therefore is dependent upon an equilibrium between loading and washout as well as the selection of a suitable seed sludge, waste water composition and careful system management. Design plays an important part in system efficiency and operational performance is strongly dependent upon effective sludge retention and activity.

OPTIMUM PERFORMANCE

For good purification results and a good process reliability at minimum investments and operation costs, the reactor must have optimum dimensions and the process must operate efficiently at

high organic loads [4]. Optimum dimensions and operation conditions can be found by means of calculations in which quantitative relations are used between the mass transports. between the sludge and the fluid in the reactor and kinetics of the biological processes in the reactor. In this connection, the following points are of interest:

- 1) The concentration of biomass in the reactor must be as high as possible. The maximum value to be reached is limited by the condition that proper operation of the settler, which includes gravity recirculation of settled sludge into the reactor, can still be guaranteed.
- 2) The biological activity of the sludge must also be as high as possible. Because it is a mixed culture, the activity strongly depends on the quantity of micro organisms in the sludge that are especially suited for conversion of the specific waste treated. Since anaerobic bacteria have a small growth rate, highly active sludge can only be obtained after long adaptation times.
- 3) The retention time of the sludge must be large, so that the appropriate mixtures of microorganism can develop during the adaptation period and can remain present in the subsequent stationary periods of operation. This can be realised by means of an effectively operating three-phase separator, and a good recirculation of settled sludge into the reactor.
- 4) The liquid residence time in the reactor must be as small as possible. In that situation the contact between the influent and the biomass must still be good. In obtaining this, two points are of importance:

- (a) the distribution system of the influent must be designed such that all the sludge in the lower parts of the bed comes into close contact with the influent, and
- b) the liquid in the bed and the blanket as well as the sludge in these areas must be mixed properly. Because mechanical mixing is not applied in these reactors, it is the turbulence resulting from the gas production that must provide proper mixing of the liquid and sludge.
- 5) The production of gas must be such that the distribution of sludge over the bed and the blanket is optimum. Since the concentration of sludge in the blanket increases with an increasing production of gas and the higher the amount of sludge in the blanket, the higher the conversion capacity in the blanket the aim is a maximum production of gas. At this maximum, the blanket sludge concentration must be such that a proper operation of the three phase separator is guaranteed. At a too high sludge concentration" in the blanket the sludge recirculation system can become blocked and the settler overloaded.
- 6) The settling properties of the sludge determine the efficiency of the settler and influence the sludge concentration in the blanket, and must therefore be optimal. By washing out sludge with inferior settling properties, which will occur when the linear fluid velocity in the reactor is high, the average settling behaviour of the sludge in the reactor will improve. Therefore, in combination with a biological adaptation time, a physical adaptation time is helpful for improvement of the settling characteristics.

7) For an optimum use of the settling properties of the sludge, it is necessary that the fluid flow pattern in the settler is laminar and uniform. This is only possible if no gas is produced in the settler, or leaks from the reactor into the settler, and if the turbulence of the liquid in the reactor does not continue in the settler. Therefore the separation of gas in the reactor must be efficient, the turbulence of the liquid in the reactor must be dampened before the liquid sludge suspension enters the settler and sludge retention time in the settler must be as small as possible.

WATER QUALITY PARAMETERS

The purification (removal) efficiency of UASB reactor can be described by using the water quality parameters [5]:

- Oxygen consuming substances
- Suspended solids
- Nutrients
- Pathogens

Removal of Oxygen Consuming Substances

Effluent values and treatment efficiencies are calculated on basis of raw (unfiltered) influent and effluent samples. It must be realised that part of the remaining pollution in the effluent consists of particulate matter (washed out biological material) and that therefore short post-settling of anaerobic effluent can further lower the effluent concentration of suspended solids, Biological Oxygen Demand (BOD) and Chemical Oxygen Demand (COD). Expressing removal efficiencies in terms of filtered effluent can thus provide an indication of the best achievable effluent quality without necessitating a more expensive microbiological polishing.

BOD removal efficiencies of typically 65-80% (dependent on the characteristics of the waste water) can be achieved in UASB reactors operated at temperatures of 20°C or higher. Generally removal efficiencies for COD are 10-20% lower than for BOD removal.

At lower sewage temperatures results are generally less encouraging. Low substrate concentrations exert a negative influence on efficiencies under unfavourable temperature conditions.

Removal of Nutrients

As a consequence of the nature of the process, anaerobic processes are unable to remove any nitrogen by means of oxidation, but organic nitrogen is converted into ammonium. Therefore anaerobic system may require aerobic nitrification as ammonium is an oxygen consuming substance. The alternative is to reuse the effluent in schemes (like irrigation) where Nitrification Oxygen Demand (NOD) is of no major concern.

Removal of Suspended Solids

High rate anaerobic reactors are characterised by relatively short hydraulic retention times which makes them more vulnerable to internal hydraulic or process related disturbances causing excessive loss of suspended material. Particularly these systems that rely on settling of suspended biomass inside the reactor, like the UASB, and systems that are designed as attached growth reactors and without any posterior settling step, like anaerobic filters, are most likely to face occasional uncontrolled washing out of suspended matter. The UASB reactors are characterised by relatively poor efficiency in suspended solids removal, with

values of 50-75% as compared to 85-95% for aerobic plants.

Anaerobic reactors applying hybrid configurations (UASB plus filter or UASB plus external sedimentation) could possibly yield better results.

CHAPTER 2

MODELLING CONCEPTS

Models have been developed to describe the removal nutrients from waste water in terms of operational variables and range from empirical relations, based on values of the operating parameters, to complex formulations based on microbiological growth kinetics. In addition to making the simplifying assumptions necessary in process modelling in general, there are a number of special problems arising in considering waste water treatment [6]. The foremost is that the two principal materials involved, the nutrients and the sludge, are both complex and variable mixtures. Nutrient composition is in most expressed in terms of the oxygen demand, and microbial concentration in terms of volatile sludge solids content. validity of their use depends on the assumption of suitable nutritional and ecological balances prevailing. In addition many modelling and experimental procedures deal with solute oxygen demand, whereas in many real systems, specially in sewage treatment processes, much of the oxygen demand is present in the form of colloidal and suspended material. While this is removed from the liquor rapidly by absorption into the sludge, complete removal is a much slower process. The engineering aspects also have to be idealized, as completely mixed and plug-flow systems are unlikely to be perfectly completely mixed or

perfectly plug flow. Liquid flow rates and oxygen demand loadings are also likely to show wide variations over a period of time. Nevertheless, models have proved themselves valuable not only in prediction of plant performances, but also in identifying key process parameters, and indicating which aspects of the process would profit most from further investigation.

Several models have been proposed to describe the overall effect of nutrient availability, as substrate concentration, on growth rate, and one that has achieved wide acceptance was proposed by Monod in 1942 [7]

$$r = \frac{K_{\text{max}} S}{K_{\text{s}} + S}$$

where $K_{\rm max}$ is the specific growth rate in conditions of unlimited nutrient availability, and $K_{\rm S}$ is a saturation coefficient characteristic of the organism and the nutrient whose availability is expressed by S. This concentration refers to the nutrient whose availability is controlling growth, and when this has the same numerical value as the saturation coefficient $K_{\rm S}$, the specific growth rate is half the maximum value. The saturation coefficient is thus sometimes called the 'half velocity constant'.

The Monod relation refers to a particular organisms growing in a system where all nutrients have unlimited availability except the controlling nutrient expressed in S. In other words, the Monod relation gives a useful description of a pure culture growing in a well defined medium. In waste treatment processes, however, the organisms and the substrates involved are both complex mixtures, and in general, relations based entirely on the Monod model have not been very successful in describing the

kinetics of BOD removal. Nutrient removal is not entirely associated with microbial growth, and the microbial ecology of sludge varies with the growth phase, so that the species found in high rate sludge are very different from those in conventional sludge. Monod based models have been most successful in describing the behaviour of specialised sludges which are more like pure cultures in character.

Since most waste treatment systems operate with the sludge in the declining, stationary or endogeneous growth phases, microbial cell decay is likely to have a significant effect on the performance of the system. Microbial cell decay is usually assumed to follow first order kinetics, so that the rate of cell decay is proportional to the concentration of residual viable cells. The coefficient of proportionately, $K_{\mbox{d}}$, is then the 'specific decay rate':

$$-\frac{dx}{dt} = K_d x$$

BIOLOGICAL REACTION KINETICS

Rarely are the complicated biological mechanisms known in. sufficient detail to allow formulation of an analytical kinetic rate expression. Certainly in waste water treatment where the biomass is a bacteriological zoo and the substrates are a mixture of household and industrial wastes, any kinetic expression for biological reaction rates must be based upon a number of simplifying assumptions. Many kinetic expressions have been formulated to fit enzyme and pure culture reactions. A few of the more common of these kinetic models are presented (Table 1) for use in modelling waste treatment systems [8].

Table 1 Common Kinetic Models

Form	Name
$K = \frac{K_{O}}{1 + K_{m}/S}$	Monod
$K = K_{O} (1 - e^{-S/K_{t}})$	·Taissier
$K = \frac{K_{O}}{1 + (K_{C} X/S)}$	Contois
$K = \frac{K_{O}}{1 + (K_{S} S^{-\lambda}M)}$	Moser
$K = \frac{K_{O}}{1 + (K_{m}/S) + (S/K_{I})}$	Haldane (Substrate inhibition)
$K = \frac{K_{O}}{1 + (K_{m}/S) + I K_{m}/K_{I}}$	Competitive inhibition
$K = \frac{K_{O}}{[1 + (K_{m}/S)][1 + (I/K_{I})]}$	Noncompetitive inhibition

BASIS FOR MODELS

Biological reactors involve a variety of geometries and hydraulic regimes. To model a biological process, we need information on the stoichiometry and kinetics of the reactions, and on the hydraulic regime of the systems. The stoichiometry of a reaction relates the quantities of reactants consumed, such as substrates, to the quantities of products formed, such as micro

organisms. The hydraulic regime refers to the patterns of flow into and out of the process, and the mixing and distribution of fluids and solids within the reactor. The influent and effluent for the process are described in terms of the time variation of flow rates and concentration of species. Any recirculation of biological solids must also be considered in the analysis.

In flow reactors, the two extremes in mixing are represented by well stirred and plug flow reactors. Intermediate degrees of. mixing are often described by well stirred reactors in series and by plug flow reactors with axial dispersion. More complex mixing models can be devised but their use may not be justified because of limitations in knowledge of the system.

CHAPTER 3

AN INTEGRAL DYNAMIC MODEL

To construct an integrated dynamic reactor model for the UASB reactor, one must know the fluid flow phenomenon, the kinetic behaviour of the microorganisms and the mass transport phenomenon between different compartments and different phases [9].

In practice, in an UASB reactor with a well functioning settler the amount of sludge steadily increases and has to be removed from time to time. This implies that the UASB reactor is hardly ever in a steady state. Theoretically, a steady state is established only when the growth of microorganisms equals the wash out. For these reasons a dynamic model is needed when the reactor behaviour has to be predicted over longer periods of time.

The mass balances for the substrate and bacterial waste products that together can be used in the description of this process can mathematically be formulated if the following is known in relation to the process conditions: the fluid flow patterns in the reactor; the distribution and behaviour of sludge in the reactor and the kinetics of the biological conversion of substrate and formation of methane and of bacterial waste products.

A number of assumptions and approximations are used in deriving the model for this reactor:

1. The chemical kinetics of the substrate and biomass reactions are described by the Monod model.

- 2. The substrate is the growth limiting substance and all other nutrients are present in excess.
- 3. The kinetic constants are independent of concentrations or the degree of conversion.
- 4. The yield coefficient (biomass formed/substrate consumed) is constant. By practice, the yield coefficient depends upon the nature of the substrate and process conditions.
- 5. The rates of the biological reactions are controlled either by chemical kinetics or by diffusional effects.
- 6. The contents of the reactor are isothermal. Since biological processes have a heat of reaction and feed conditions may change, temperature variations are possible. In most waste water treatment reactors, short term temperature changes are usually small.
- 7. Physical properties of the fluid are constant. If the average values are used, little error is introduced.
- 8. The transport of substrate through the fluid is rapid relative to the rate of reaction so that concentration gradients in the bulk liquid are negligible.

The UASB reactor can be divided in three compartments: the sludge bed, the sludge blanket, and the internal settler.

In studies on the fluid flow pattern in upflow reactors, it was found that both the sludge bed and the sludge blanket can be described as separate well-mixed flow regions. Consequently, in the description of the fluid flow pattern, two ideal mixers will be taken for these two areas.

Furthermore bypassing stream (Q_k) of part of the influent along the bed to the blanket is assumed. The bed and blanket

regions are interconnected by exchange streams $(Q_{\rm bd}$ and $Q_{\rm db})$; and that no dead space is assumed to be present; where Q is influent flow rate. (as shown in Fig. 1.)

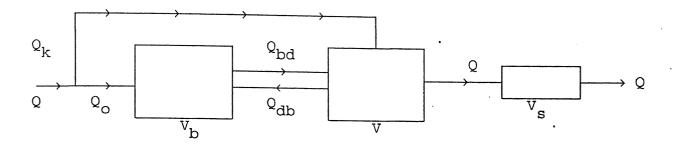


Fig. 1: Compartments of a UASB Reactor with bypassing and intermixing streams.

For this model, the following relations hold:

$$V_{R} = V_{b} + V_{d} + V_{pf}$$
 (1)

$$Q = Q_0 + Q_k$$
 (2)

$$Q_{bd} = Q_o + Q_{db}$$
 (3)

where, $V_{\rm p}$ is total volume of reaction

 $V_{\rm b}$ is volume of bed

 $V_{\rm d}$ is volume of blanket

and V_s is volume of settler

In analogy with the distribution of liquid over two main regions in the reactor, the sludge is distributed over a sludge bed and a sludge blanket. Because of the small growth rate of methane forming bacteria the washout of sludge must be minimum to have atleast a constant, or more preferable, a slowly increasing amount of sludge in the reactor.

The equations used for the kinetics of the substrate conversion are analogous to the Monod equations for biological.

growth. Production of bacterial waste is assumed to be proportional to the concentration of substrate.

It is now further assumed that both in the bed and in the blanket substrate is converted into gas and bacterial waste products, while in the bypassing channels through the bed no conversion is assumed to take place.

In the following the relevant constituents are discussed for each compartment.

Bed

Substrate

The mass balance for the substrate over the bed has to be formulated as follows. In a well mixed reactor the composition is uniform throughout the reactor. Thus the exit stream from this type of reactor has the same composition as the fluid within the reactor. The mixing action must be sufficient to disperse the incoming feed rapidly throughout the reactor.

A schematic diagram of a well mixed biological reactor of volume $\mathbf{V}_{\mathbf{b}}$ is shown in Fig. 2.

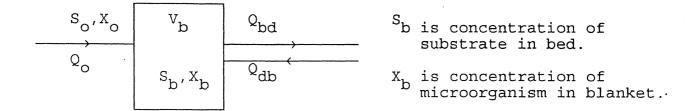


Fig. 2: Schematic diagram of the Bed in a UASB Reactor.

The stream arriving at the process has a flow rate Q_0 , a substrate concentration S_0 , and a biomass concentration $X_0 = 0$.

A material balance around the reactor states:

Accumulation = Input - Output + Formation by Reaction.

For the substrate, the material balances are

$$^{V}_{b} \frac{dS_{b}}{dt} = Q_{o} S_{o} + Q_{db} S_{d} - Q_{bd} S_{b} - r_{b} V_{b}$$
 (5)

If the rate of reaction follows the Monod kinetic model,

$$r_b = \frac{K_{\text{max}} S_b X_b}{K_S + S_b}$$
 (6)

Substituting this rate expression, the material balance becomes

$$V_{b} \frac{dS_{b}}{dt} = Q_{o} S_{o} + Q_{db} S_{d} - Q_{bd} S_{b} - K_{max} \frac{S_{b} X_{b}}{K_{s} + S_{b}} V_{b}$$
 (7)

Microorganism

For the microorganism, the material balance is:

$$V_{b} \frac{dX_{b}}{dt} = Q_{db} X_{d} - Q_{bd}X_{b} + r_{x} V_{b}$$
(8)

If the rate of reaction is proportional to the concentration of substrate and the rate of decay is proportional to the concentration of microorganism, then

$$r_{x} = K_{x} S_{b} - K_{d} X_{b}$$
 (9)

Substituting the rate expression, the material balance becomes

$$V_{b} \frac{dX_{b}}{dt} = Q_{db} X_{d} - Q_{bd} X_{b} + (K_{x} S_{b} - K_{d} X_{b}) V_{b}$$
 (10)

where $\mathbf{K}_{\mathbf{X}}$ is microorganism growth constant and $\mathbf{K}_{\mathbf{d}}$ is microorganism decay constant.

Blanket

These relations are analogous to their counterparts in the sludge bed.

Substrate

$$V_{d} \frac{dS_{d}}{dt} = Q_{k} S_{o} - Q_{bd} S_{b} - Q_{db} S_{d} - K_{max} \frac{S_{d} X_{d}}{K_{s} + S_{d}} V_{d}$$
 (11)

Microorganism

$$V_{d} \frac{dX_{d}}{dt} = Q_{bd}X_{b} - Q_{db}X_{d} - QX_{d} + (K_{x} S_{d} - K_{d} X_{d}) V_{d}$$
 (12)

Thus we get a stiff system of four first order ordinary. differential equations (7), (10), (11) & (12). We substitute the various values of V_b , V_d , Q_k , Q_o , Q_b , Q_{bd} , Q_{db} , Q_{db} , Q_{max} , $Q_$

The initial conditions used are at

- (i) The variation of s_b , s_d , x_b , x_d with time was obtained by solving the four differential equations.
- (ii) The variations of S_b , S_d , X_b , X_d with change in one of the input variables Q_{bd} , Q_{db} , Q_o , Q_k , X_x , X_{max} , X_s and X_d was studied.

The range of variables studied are given in Table 2.

Table - 2: Range of Variables

Sl No	K _d	K _{max}	K s	K _x	Qo	Q _k	Q _{db}	Q _{bd}
1	0	0	0	0	2	0	0	1.9
2	.002	. 5	4	.01	1.9 ·	0.1	0.1	2
3.	.02	1	40	.1	1.5	0.5	0.5	2.4
4.	. 2	2	100	1	1 .	1	1	2.9
5	2	5	1000	10	0.5	1.5	2	3.9

(iii) The concentration of substrate in bed and blanket was studied when a part of the sludge is withdrawn, as the microorganism concentration increases above a fixed limit. The concentration of microorganism is made equal to the microorganism concentration at t=0.

CHAPTER 4

RESULTS AND DISCUSSION

- A. General behaviour of Substrate and Microorganism concentration with time.
- (i) Variation of substrate concentration in Bed and Blanket with time:

The base values are:

$$K_{d} = .02$$
, $K_{max} = 1$, $K_{s} = 40$, $K_{x} = .1$, $Q_{k} = 0.1$, $Q_{o} = 1.9$, $Q_{bd} = 2$, $Q_{db} = .1$.

The concentration of substrate in the bed and blanket decreases with time.

In the blanket the substrate concentration is lower than in the bed. This is because the influent stream in the blanket is of a lower substrate concentration (as effluent from the bed) causing low substrate concentration in the blanket.

(ii) Variation of microorganism concentration in Bed and Blanket with Time:

The concentration of microorganism in the bed and blanket first increases and then decreases with time. This is because initially the substrate concentration is higher and the growth is more than the decay. Later on the concentration of substrate diminishes and that reduces microorganism concentration. The results obtained are presented in Table 3.

Table 3:General Behaviour of Substrate and Micro-organism Concentration with Time

t .	Sb	Sd	Xb	Xd
0	500	500	60	60
l	433	424	93	103
2	355	318	114	136
3	277	199	124	155
4	209	88	126	161
5	156	20	122	158
6	120	5	115	151
7	100	4	106	144
8	91	4	98	136
9	90	4	90	129
10	93	4	84	122
11	98	4	79	115
12	104	· 5	76	109

B. Variation of substrate and Microorganism concentration with change in one of the input variables:

(i) Variation of Decay Constant (K_d) :

It is observed that with increase in decay constant, there is a sharp reduction in concentration of microorganisms. At the same time with low concentration of microorganisms, the conversion of substrate also gets reduced.

When $K_{\mbox{d}}=0$ there is no decay of microorganisms, so the concentration of microorganisms increases with time, reaches a maximum and then decreases. The substrate concentration also reaches a low value, because of the high concentration of microorganisms. The changes in substrate and microorganism concentration profiles as a function of time with variation in decay constant is shown in Figs. 3 and 4.

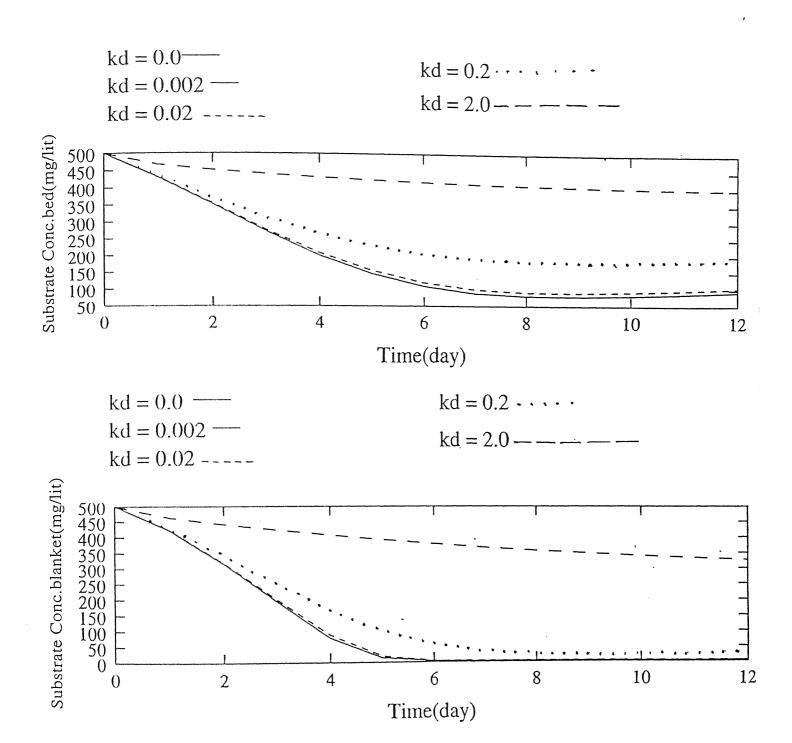


Fig.3: Substrate Concentration Profiles with Variation in Decay Constant

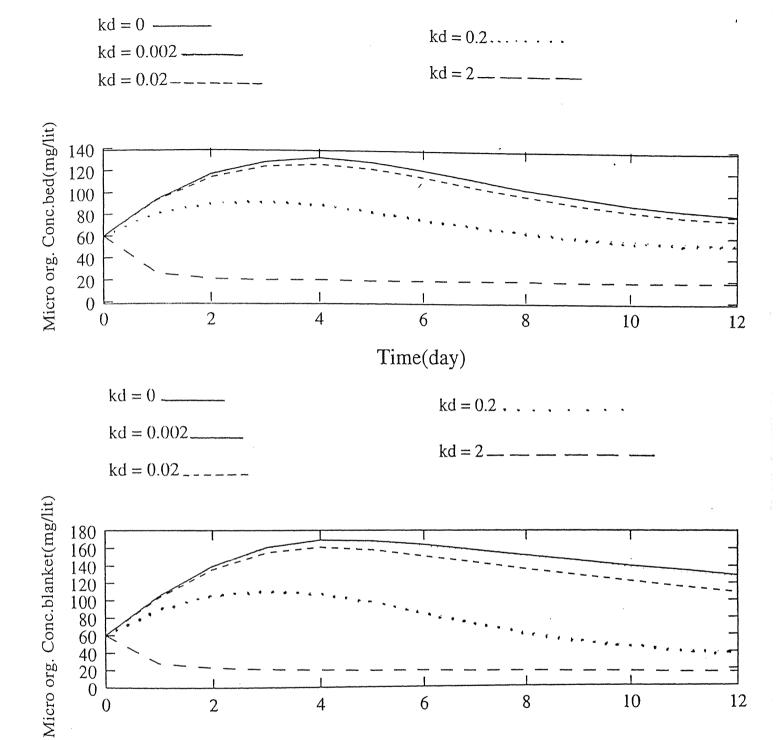


Fig. 4: Microorganism Concentration Profiles with Variation in Decay Constant

Time(day)

(ii) Variation of Monod Kinetic Constant (K_{max}) :

For $K_{\text{max}} = 0$, there would not be any reduction in substrate concentrations, as the rate of conversion of substrate from Monod Kinetics becomes zero, and the microorganism concentration increases.

For high values of K_{max} the reaction becomes fast and the substrate gets reduced completely. There is a decrease in microorganims concentration. The decrease in microorganism concentration is more in the blanket than in the bed.

The variations in K_{max} has a considerable effect on the concentration profiles, as can be seen in Figs. 5 and 6.

(iii) Variation of Saturation Coefficient (K_S) :

For $K_{\rm S}=0$, the rate of substrate conversion is dependent only on microorganism concentration. The substrate concentration finally reduces to zero.

For large values of $K_{\rm S}$ the substrate conversion decreases and microorganisms increases, as shown in Figs. 7 and 8.

(iv) Variation in K_x :

For $K_{\rm x}$ = 0 and low $K_{\rm x}$ values the growth of microorganism is slow and the concentration of microorganism reduces because of decay. With low microorganism concentration, the conversion of the substrate also reduces.

For high $K_{_{\mbox{\scriptsize X}}}$ values of microorganisms grow fast, as a result of which the substrate concentration is also low, as shown in Figs. 9 and 10.

(v) Variation in Q_{bd} and Q_{db} (Exchange streams):

The value of flow streams $Q_{\mbox{bd}}$ and $Q_{\mbox{db}}$ are related by equation

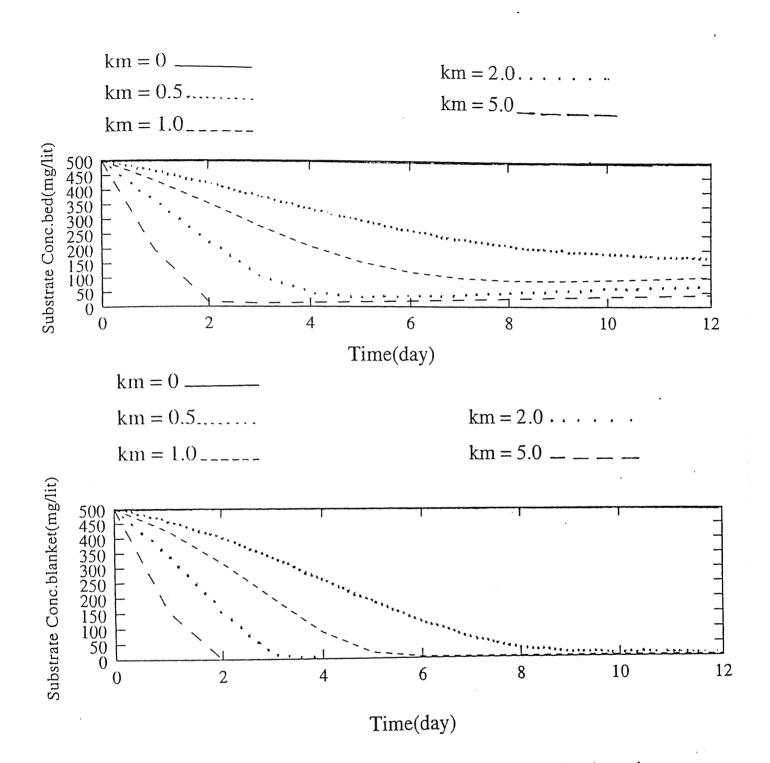


Fig.5: Substrate Concentration Profiles with Variation in Monod Kinetic Constant

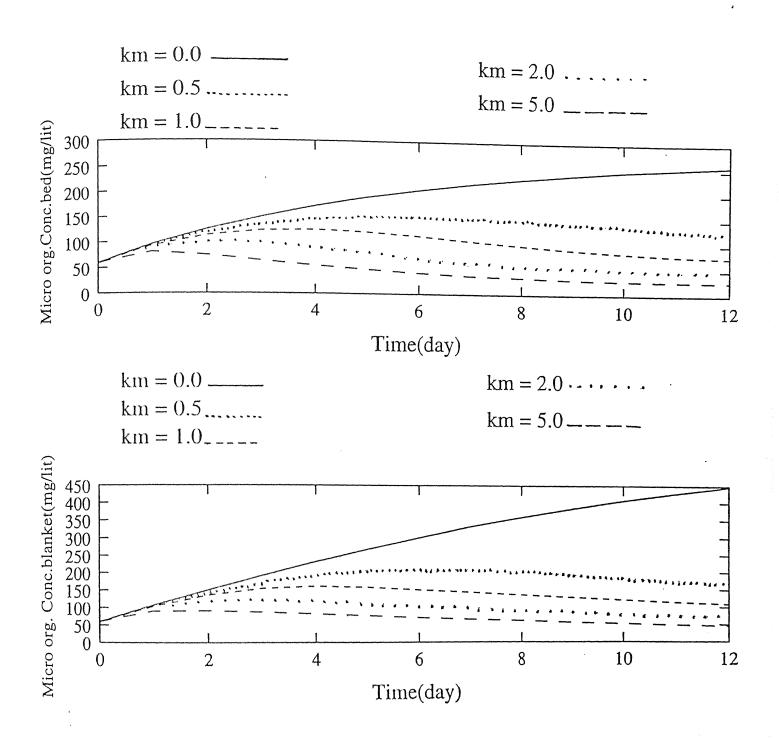


Fig. 6: Microorganism Concentration Profiles with Variation in Monod Kinetic Constant

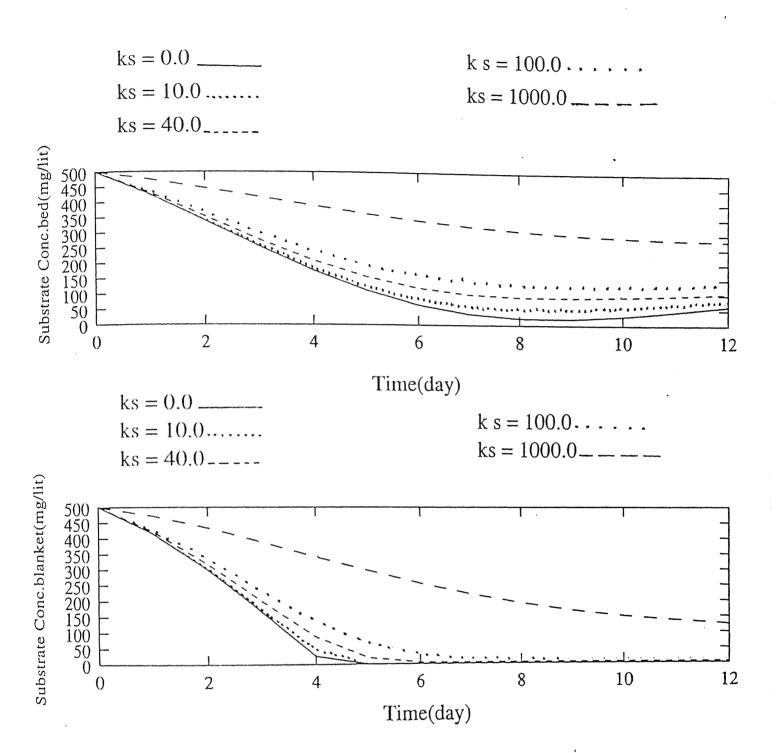
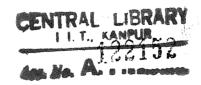


Fig. 7: Substrate Concentration Profiles with Variation in Saturation Coefficient



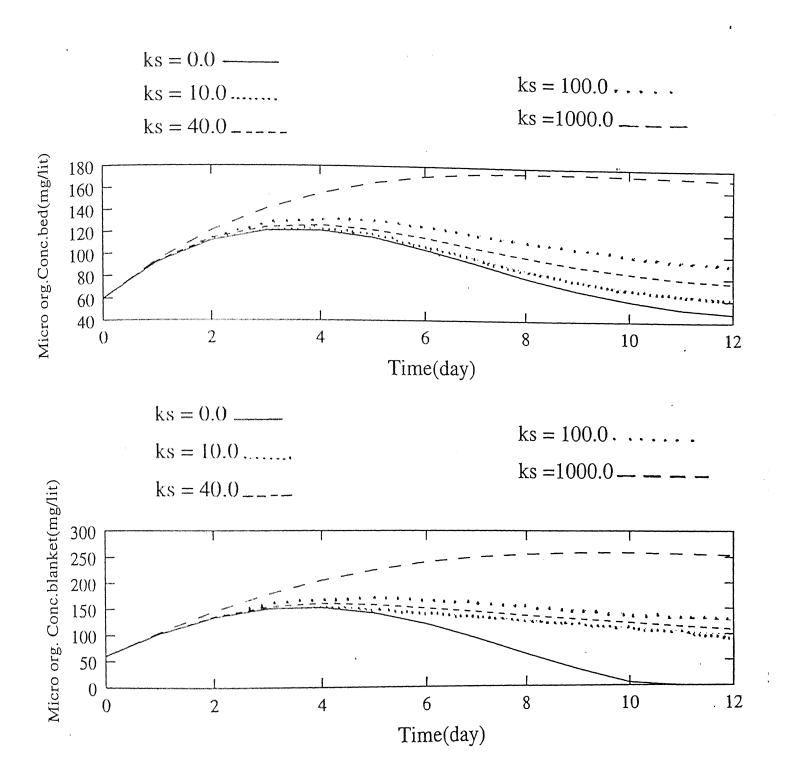


Fig.8: Microorganism Concentration Profiles with Variation in Saturation Coefficient

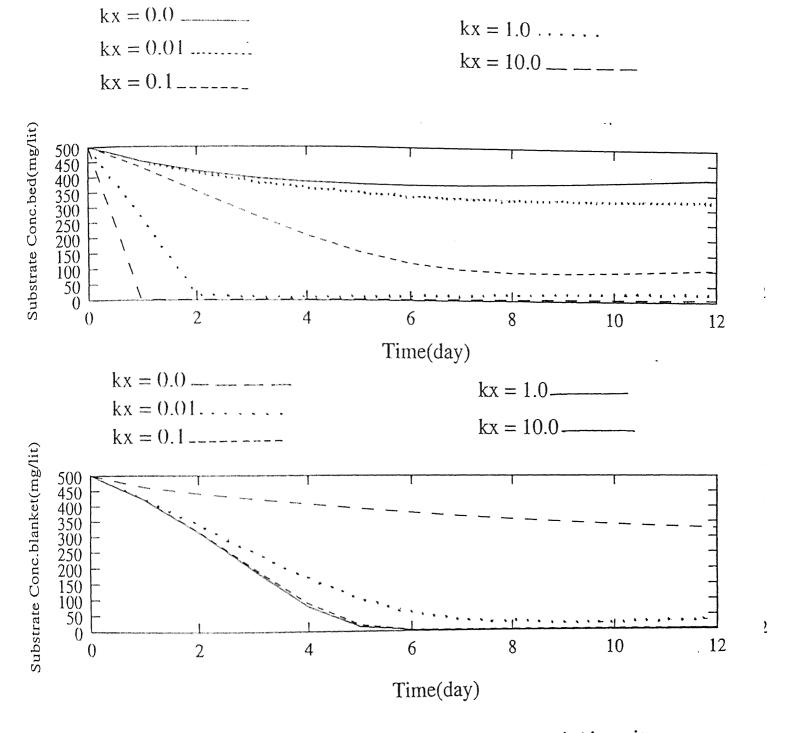


Fig.9: Substrate Concentration Profiles with Variation in Microorganism Growth Constant

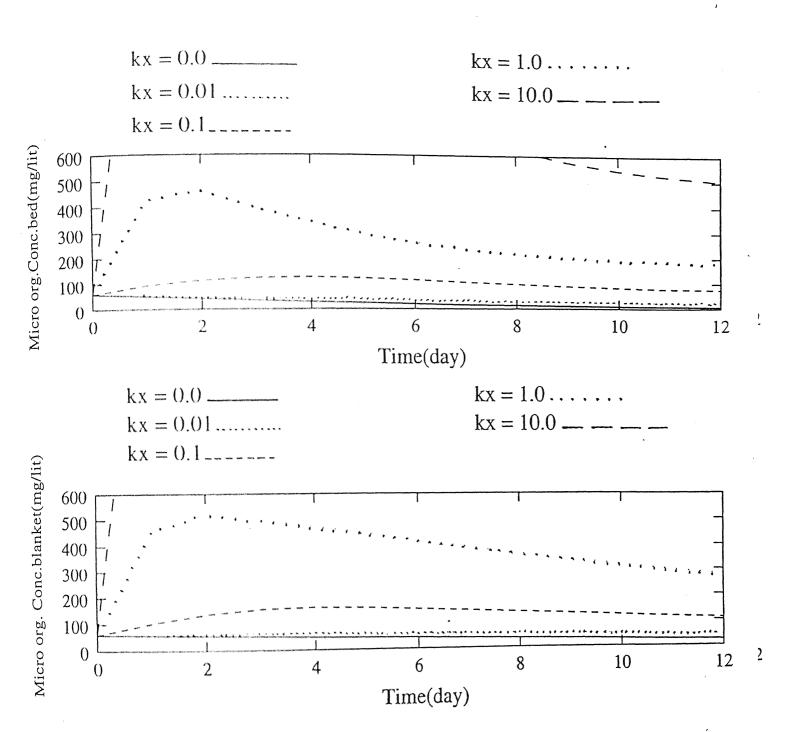


Fig. 10: Microorganism Concentration Profiles with Variation in Microorganism Growth Constant

3 and since $Q_{\rm O}$ is constant at 1.9 m³/hr, the change in one stream will change the other stream also. Increase in the value of $Q_{\rm db}$ will decrease the substrate concentration in bed and blanket thus increasing the conversion. This can be seen in Figs. 11 and 12.

(vi) Variation in Q_0 and Q_k (By passing streams):

When $Q_k=0$, the substrate conversion in the bed decreases. This is because of the increase in flow rate of influent in the bed, causing reduction in the residence time in the reactor. The microorganism concentration in the blanket is more as compared to that in the bed.

When Q_k increases, the substrate concentration in the blanket increases, while that in the bed decreases. The microorganism concentration in the bed decreases, while that in the blanket increases as shown in Figs. 13 and 14. If Q_k is increased beyond a limit, there is no reduction in substrate concentration in the blanket, and the microorganism in the blanket also die out.

C. Variation in substrate and Microorganism concentration on sludge withdrawal

The dynamic model shows that in about 10 days operation, the substrate concentration has been reduced to a very low value but the microorganism concentration is high. This necessitates the removal of sludge from the reactor periodically so that a near stable steady state operation can be carried out. In this problem, the microorganism concentration at the start of the reactor was 60 mg/l. It is assumed now that the maximum concentration of microorganism in the bed and blanket will not be allowed to

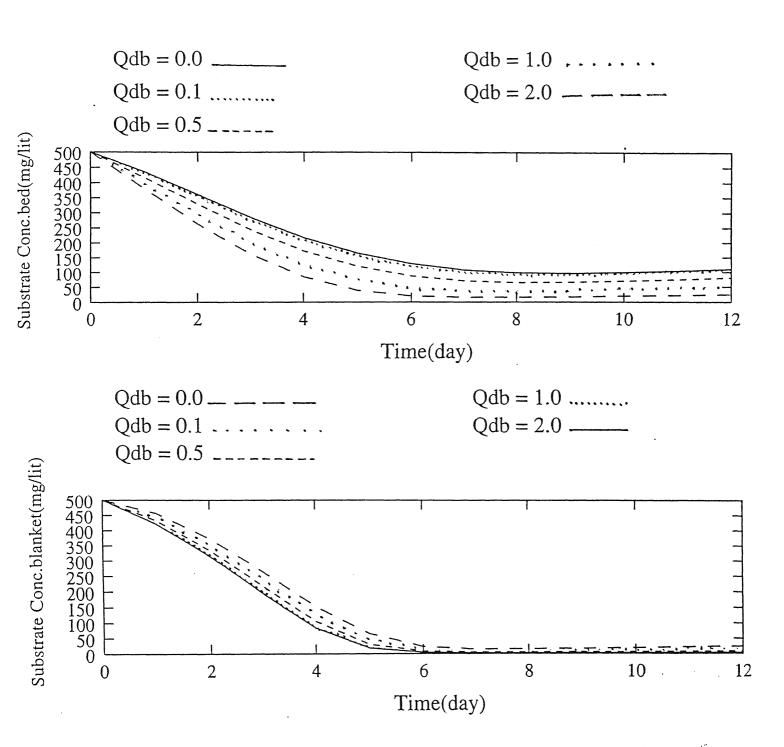


Fig. 11: Substrate Concentration Profiles with Variation in Exchange Streams

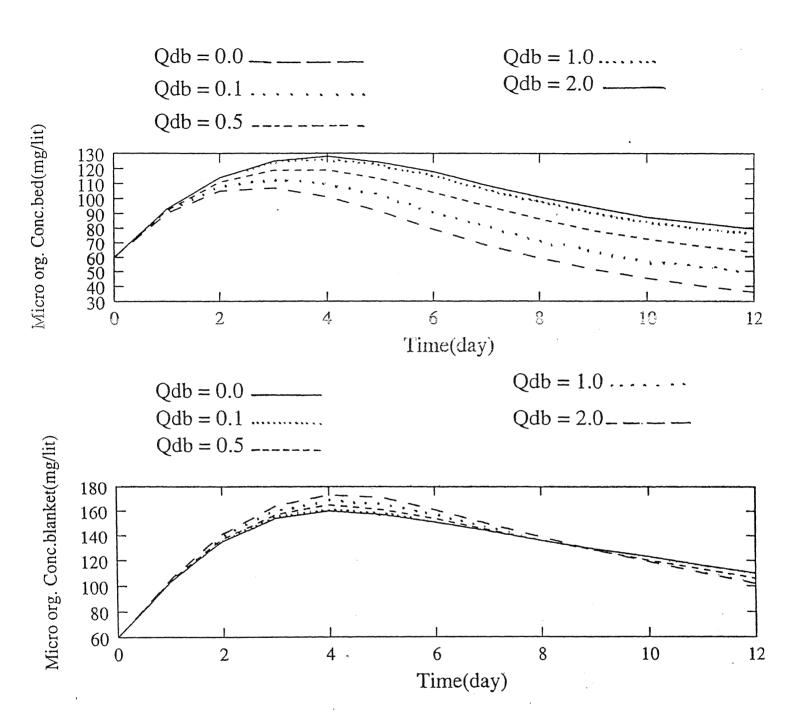


Fig. 12: Microorganism Concentration Profiles with Variation in Exchange Streams

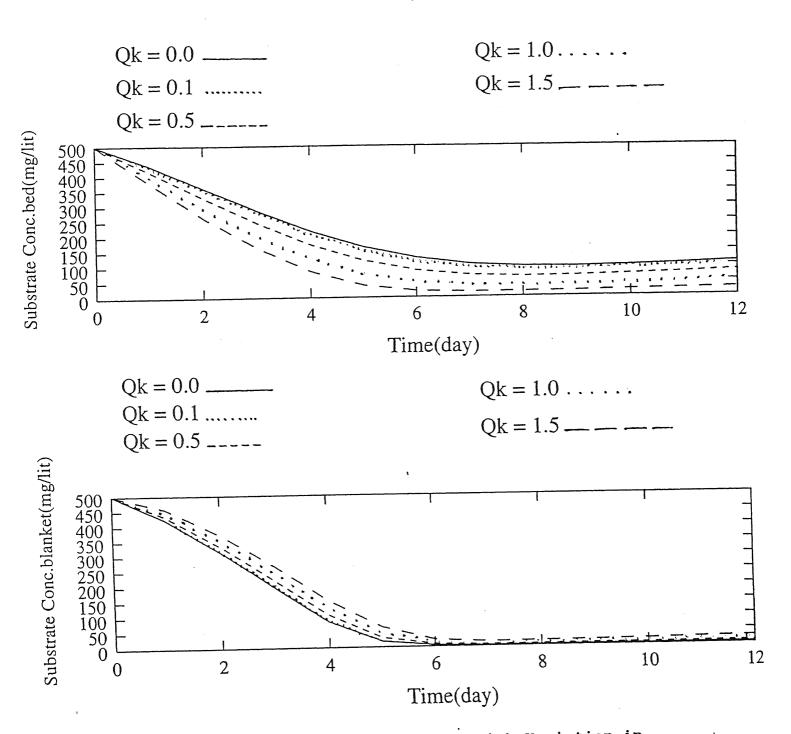


Fig.13: Substrate Concentration Profiles with Variation in bypassing Stream

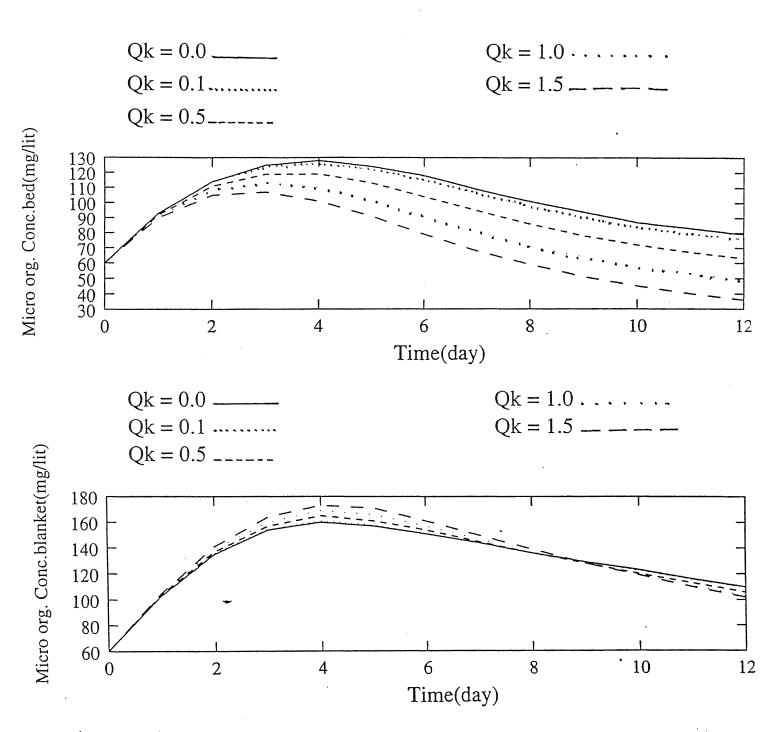


Fig.14: Microorganism Concentration Profiles with Variation in bypassing Stream

exceed 100 mg/l and so when it reaches 100 mg/l, the sludge will be removed to such an extent that its concentration is reduced to 60 mg/l. This is done by making suitable modifications in the original NAG Fortran Library Routine. The results obtained are presented in Table 4.

Table 4: Variation in Substrate and Microorganism Concentration on Sludge withdrawal

t	Sb	Sd	Xb	Xd
0	500	500	60	60
1	433	424	93	60
2	355	352	60	99
3	317	257	81	60
1	271	204	94	83
5	225	140	60	98
6	213	80	70	60
7	196	58	77	66
8	178	38	82	70
9	162	25	84	73
10	148	17	84	74
11	138	13	83	75
12	131	11	82	75

CHAPTER 5

CONCLUSIONS

- An upflow anaerobic sludge blanket reactor can be modelled as two mixed reactors in series with by passing and intermixing streams.
- 2. The substrate concentration $(S_{\rm b} \ {\rm and} \ S_{\rm d})$ decreases with time and reaches a steady state value. The most sensitive parameter effecting $S_{\rm b}$ and $S_{\rm d}$ is Monod kinetic constant $k_{\rm max}$.
- 3. For empty bed hydraulic retention time of 15 hours, and the constants in the range studied (Table 2), it takes 5 to 8 days to reach steady state value for substrate concentration.
- 4. The microorganism concentrations $(X_b$ and $X_d)$ first increases and then decreases with time. The most sensitive parameter effecting X_b and X_d is decay constant k_d .
- Except for high values of $k_{\mbox{d}}$ ($k_{\mbox{d}}$ > 0.02), the microorganism concentration in the bed and blanket increases to a high value, necessitating withdrawal of sludge from the reactor from time to time.
- 6. Comparing the results for sludge concentration in the two cases, i.e. i) when sludge is not removed from the reactor (Table 3) and (ii) when sludge is removed as the sludge concentration reaches 100 mg/l (Table 4), it can be said that sludge has to be removed once or twice in a day.

CHAPTER 6

RECOMMENDATIONS

The model has been studied considering both bed and blanket as mixed reactors. To extend the study, the bed can be considered as a mixed reactor and the blanket as a plug flow reactor.

The variation of following parameters which have not been studied i.e. $V_{\rm b}^{}$, $V_{\rm d}^{}$, Q can be studied.

The effect of volume change due to the growth of microorganism and decay of microorganism can be considered.

NOMENCLATURE

 $K_d = Decay constant, day^{-1}$

 $K_{max} = Monod Kinetic Constant, day^{-1}$

 K_{x} = Microorganism Growth Constant, day⁻¹

 $K_{S} = Saturation Coefficient, mg/1$

Q = Influent Flow Rate, m^3/day

 $Q_{\rm bd}$ = Flow Rate of Exchange stream from bed to blanket, m^3/day

 $Q_{\rm db}$ = Flow Rate of Exchange stream from blanket to bed, $m^3/{\rm day}$

 Q_k = By passing stream Flow Rate, m^3/day

 $S_{O} = Substrate concentration at time t = 0, mg/1$

 $S_b = Substrate concentration in bed, mg/l$

 $S_d = Substrate concentration in blanket, mg/l$

V = Volume of reactor, m³

 $V_b = Volume of bed, m³$

 $V_d = Volume of blanket, m³$

 $V_s = Volume of settler, m³$

 X_{b} = Microorganism concentration in bed, mg/l

 X_{d} = Microorganism concentration in blanket, mg/1

t = Time, day

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APPENDIX

```
D02EBF Example Program Text
   Mark 14 Revised. NAG Copyright 1989.
   .. Parameters ..
   COMMON\ /yash/Vb, Vd, Q, Qdb, Qbd, Qo, Qk, So, Kx, Kmax, Ks, Kd
   DOUBLE PRECISION Vb. Vd, Q, Qdb, Qbd, Qo, Qk, So, Kx, Kmax, Ks, Kd
   INTEGER
                  NOUT
   PARAMETER
                    (NOUT=6)
                 N, IW
   INTEGER
   PARAMETER
                    (N=4,IW=(12+N)*N+50)
   .. Scalars in Common ..
   DOUBLE PRECISION H, XEND
   INTEGER
                 T
   .. Local Scalars ..
   DOUBLE PRECISION TOL, X
   INTEGER
                 IFAIL, IR, MPED
  .. Local Arrays ..
   DOUBLE PRECISION W(IW), Y(N)
  .. External Subroutines ..
   EXTERNAL
                   D02EBF, D02EJY, FCN, OUT, PEDERV
   .. Intrinsic Functions ..
   INTRINSIC DBLE
   .. Common blocks ..
                 XEND, H, I
   COMMON
   .. Executable Statements ..
   READ *, Vb, Vd, Q, Qdb, Qbd, Qo, Qk, So, Kx, Kmax, Ks, Kd
   WRITE (NOUT.*) 'Vb Vd Q Qdb Qbd Qo Qk So Kx Kmax Ks Kd'
   WRITE (NOUT, 999) Vb, Vd, Q, Qdb, Qbd, Qo, Qk, So, Kx, Kmax, Ks, Kd
999 FORMAT (2i4,5f4.1,2x,i3,2x,f6.3,2i6,2x,f6.3)
   WRITE (NOUT,*) 'D02EBF Example Program Results'
   WRITE (NOUT,*)
   WRITE (NOUT,*) 'Calculating Jacobian internally'
   MPED = 0
   IR = 1
   XEND = 12
     TOL = 10.0D0**(-3)
     WRITE (NOUT,*)
     WRITE (NOUT,99999) ' Calculation with TOL =', TOL
     WRITE (NOUT,*) 'X Y(1) Y(2) Y(3) Y(4)'
     X = 0.0D0
     Y(1) = 500
     Y(2) = 500
     Y(3) = 60
        Y(4) = 60
     I = 11
     H = (XEND-X)/DBLE(I+1)
     IFAIL = 0
     CALL D02EBF(X,XEND,N,Y,TOL,IR,FCN,MPED,D02EJY,OUT,W,IW,IFAIL)
```

```
IF (TOL.LT.0.0D0) WRITE (NOUT,*)' Range too short for TOL'
            WRITE (NOUT,*)
            WRITE (NOUT,*) 'Calculating Jacobian by PEDERV'
                TOL = 10.0D0**(-3)
                WRITE (NOUT,*)
                WRITE (NOUT,99999) 'Calculation with TOL =', TOL
                WRITE (NOUT,*) 'X Y(1) Y(2) Y(3) Y(4)'
               X = 0.0D0
               Y(1) = 500
               Y(2) = 500
               Y(3) = 60
                    Y(4) = 60
               I = 11
              H = (XEND-X)/DBLE(I+1)
               IFAIL = 0
               CALL D02EBF(X,XEND.N.Y,TOL,IR,FCN,MPED,PEDERV,OUT,W,IW,IFAIL)
              IF (TOL.LT.0.0D0) WRITE (NOUT,*)' Range too short for TOL'
99999 FORMAT (1X,A,D8.1)
       SUBROUTINE FCN(T,Y,F)
    .. Parameters ..
       INTEGER
       PARAMETER (N=4)
     .. Scalar Arguments ..
       DOUBLE PRECISION T
        COMMON /yash/Vb, Vd.Q.Qdb,Qbd,Qo,Qk,So,Kx,Kmax,Ks,Kd
        DOUBLE PRECISION Vb.Vd.Q.Qdb.Qbd.Qo,Qk,So,Kx,Kmax,Ks,Kd
        .. Array Arguments ..
        DOUBLE PRECISION F(N), Y(N)
       WRITE (NOUT,*) 'ENTERED THE function fcn '
      WRITE (NOUT,*) 'ENTER THE VALUES Vb, Vd,Q,Qdb,Qbd,Qo,Qk,So,Kx,
                                 Kmax, Ks, Kd'
      .. Executable Statements ..
       F(1) = Qo*So/Vb + Qdb/Vb*Y(2) - Qbd/Vb*Y(1) - Kmax*Y(3)*Y(1)/Vb*Y(1) - Kmax*Y(3)*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1) - Kmax*Y(3)*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1)/Vb*Y(1
                         (Ks+Y(1))
        F(2) = Qk*So/Vd + Qbd/Vd*Y(1) - Qdb/Vd*Y(2) - Q/Vd*Y(2) -
       + Kmax*Y(4)*Y(2)/(Ks+Y(2))
        F(3) = Qdb/Vb*Y(4) - Qbd/Vb*Y(3) + Kx*Y(1)-Kd*Y(3)
        F(4) = Qbd/Vd*Y(3) - (Qdb+Q)/Vd*Y(4) + Kx*Y(2)-Kd*Y(4)
        RETURNO
        END
        SUBROUTINE PEDERV(X,Y,PW)
       .. Parameters ..
        INTEGER
                                                   (N=4)
        PARAMETER
       .. Scalar Arguments ..
        DOUBLE PRECISION X
```

```
.. Array Arguments ..
                DOUBLE PRECISION PW(N,N), Y(N)
                 .. Executable Statements ..
                 COMMON \ / yash/Vb, Vd, Q, Qdb, Qbd, Qo, Qk, So, Kx, Kmax, Ks, Kd
                DOUBLE PRECISION Vb, Vd, Q, Qdb, Qbd, Qo, Qk, So, Kx, Kmax, Ks, Kd
                PW(1,1) = -Qbd/Vb-Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)*Y(1)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+Kmax*Y(3)/(Ks+Y(1))+K
               + ((Ks+Y(1))**2)
                PW(1,2) = Qdb/Vb
                PW(1,3) = -Kmax*y(1)/(Ks+Y(1))
                PW(1,4) = 0
                PW(2,1) = Qbd/Vd
                PW(2,2) = -Qdb/Vd - Q/Vd - Kmax * Y(4)/(Ks + Y(2)) + Kmax * Y(4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4) * (4)
                                 Y(2)/((Ks+Y(2))**2)
               PW(2,3) = 0
               PW(2,4) = -Kmax*Y(2)/(Ks+Y(2))
               PW(3,1) = K_X
               PW(3,2) = 0
               PW(3,3) = -Qbd/Vb -Kd
               PW(3,4) = Qdb/Vb
              PW(4,1) = 0.0
              PW(4,2) = +K_X
              PW(4.3) = Qbd/Vd
              PW(4.4) = -(Qdb+Q)/Vd-Kd
              RETURN
              END
              SUBROUTINE OUT(X,Y)
             .. Parameters ..
              INTEGER
              PARAMETER (N=4)
              INTEGER
                                                                   NOUT
              PARAMETER (NOUT=6)
           .. Scalar Arguments ..
              DOUBLE PRECISION X
           .. Array Arguments ..
              DOUBLE PRECISION Y(N)
              .. Scalars in Common ..
              DOUBLE PRECISION H, XEND
              INTEGER
                                                                T
              .. Local Scalars ..
              INTEGER J
              .. Intrinsic Functions ..
             INTRINSIC DBLE
              .. Common blocks ..
              COMMON XEND, H, I
                .. Executable Statements ..
              WRITE (NOUT,99999) X, (Y(J),J=1,N)
              X = XEND - DBLE(I)*H
              I = I - I
              RETURN
99999 FORMAT (1X,i8,4i13)
```

END